Weibull – When Being Normal Is Not Good Enough

Dennis Keisic's Legacy:

Using the Six Sigma Process and Nomenclature for Non-normal Distributions

Presented by Wes Fulton, CEO of Fulton Findings
And Carl Tarum, President of Bathtub Software LLC
For ASQ Reliability & Risk Division, 9 JUL 2020

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Beginnings

- AUG 2010 – Meeting w/ Dennis Keisic at GM HQ, Renaissance Center, Detroit, MI
- Dennis asks to put Quality Limit Comparison (QLC) into software using his "PPM equivalence" method
- NOV 2010 – His QLC option is included in the SuperSMITH® Weibull software
  - Ppk output implemented for simplicity of input
  - Cpk (TBD) requires extra sequencing input
  - PPM equivalence idea is the same for both

Weibull and Six Sigma

Agenda

(1) 1st Six Sigma efforts with normal "bell-curve" distribution exclusively

(2) Look at the normal distribution

(3) Evaluate "Almost Normal" Data

(4) Assembly Plant Capability Study of Cardan Joint

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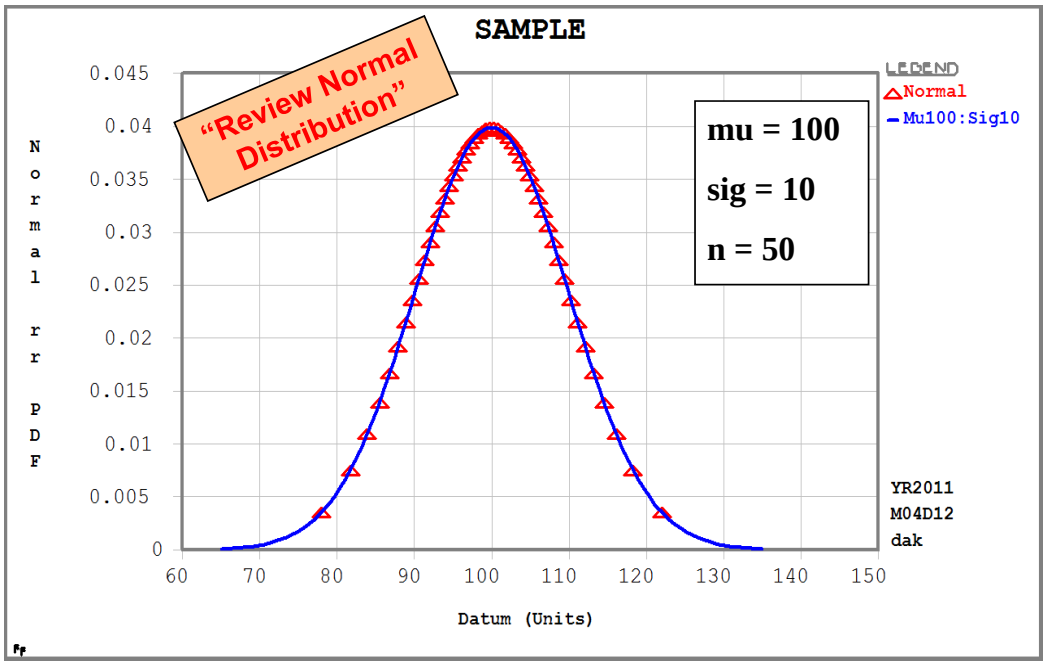
Normal Distribution



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Normal Distribution

$$P_{pu} = \frac{USL - mu}{3\ sig}$$

$$P_{pl} = \frac{mu - LSL}{3\ sig}$$

Ppk = Process Capability u = Upper, l = Lower

(Ppk is either Ppl or Ppu, whichever is smaller)

USL = Upper Statistical Limit
LSL = Lower Statistical Limit
mu = x bar or average

sig = Standard Deviation or sigma

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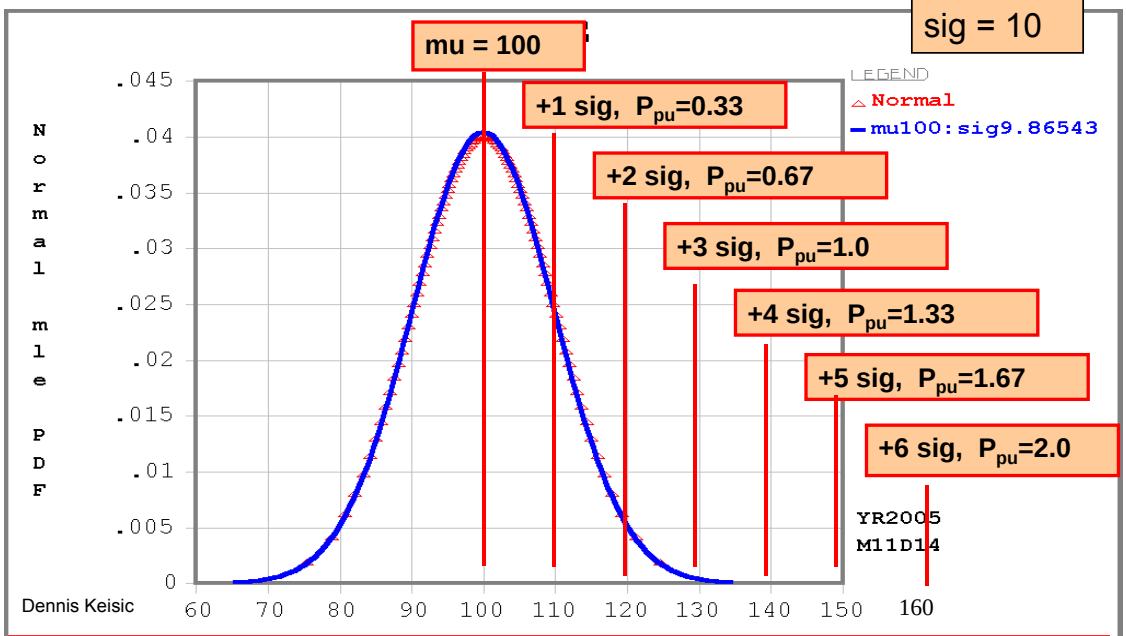
Normal Distribution

mu = 100
sig = 10



If USL = +3sig, then P_{pu}=1.0; if USL = +5sig, then P_{pu}=1.67; etc.

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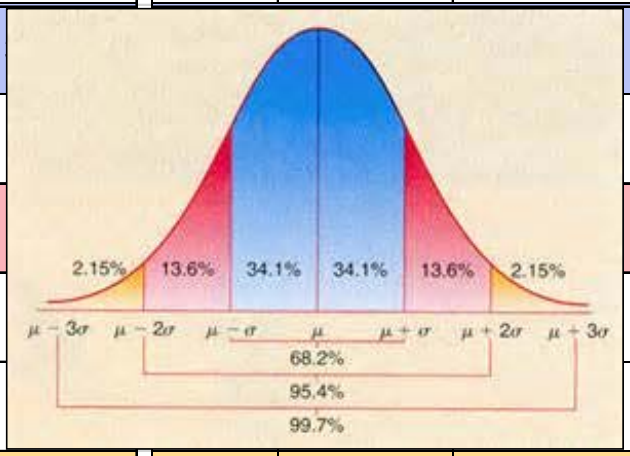
Normal Distribution

Comparison: P_p / Sigma / Double Sided PPM / Single Sided PPM / Weibull							
Ppk	Sigma	Double Sided		Single Sided			
		Inside Bounds	PPM	PPM	Lower Tail	Upper Tail	PPM DPMO
0.33	1	68.27%					158,650
0.50	1.5	86.64%					66,800
0.67	2	95.45%					22,750
0.77	2.33	98.00%					10,000
0.83	2.5	98.76%					6,200
1.00	3	99.73%	2,700	1,350	0.135%	99.865%	1,350



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Normal Distribution

Comparison: P_p / Sigma / Double Sided PPM / Single Sided PPM / Weibull							
Ppk	Sigma	Double Sided		Single Sided			
		Inside Bounds	PPM	ppm	Lower Tail	Upper Tail	PPM DPMO
1.00	3	99.730%	2,700	1,350	0.135%	99.865%	1,350
1.17	3.5	99.9535%	465	233	0.023%	99.9767%	233
1.33	4	99.9937%	63	32	0.003%	99.9968%	32
1.42	4.27	99.9980%	20	10	0.001%	99.9990%	10
1.50	4.5	99.999320%	6.8	3.4	0.000340%	99.99966%	3.4
1.67	5	99.999943%	0.6	0.3	0.00003%	99.9999997%	0.3
2.00	6	99.9999998%	0.002	0.001	0.0000001%	99.9999999%	0.001

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Non-Normal Data Sources

- Maybe only positive (but Normal predicts some zero and some negative)
- Maybe not symmetrical around central value (but Normal is symmetrical)

… Manufacturing / Performance …	Run out
Backlash	Straightness
Balance (Imbalance)	Surface finish
Case depth	Tool wear
Corrosion	Torque
Leak test (Water drop / Pressure)	True position (drilled hole)
Life data / Vehicle / Component	
Perpendicularity	… Other …
Plating thickness	Customer waiting time
Press fit	Time to pay bills

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Non-Normal Distribution

So-called “Standard” Six Sigma process capability calcs use the “Normal” model for analysis.

- - - Versus a <u>possible</u> “*Free Lunch*” with Weibull Engineering (“WE”) … including appropriate alternatives.

- Let data determine the most appropriate model.

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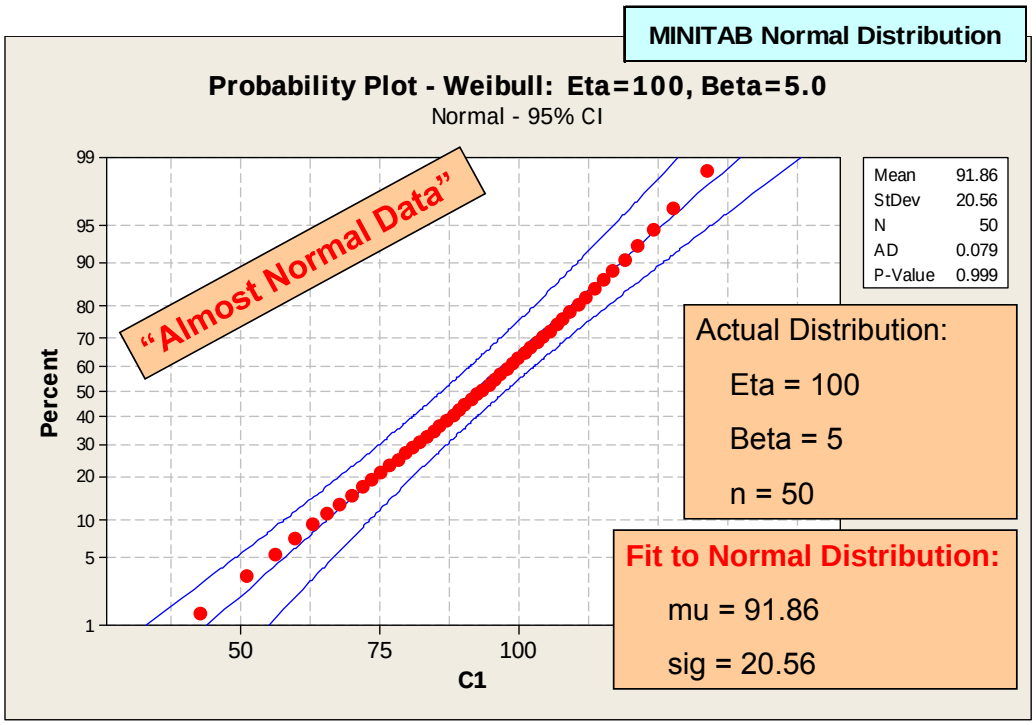
Weibull Data Analyzed with Normal Distribution



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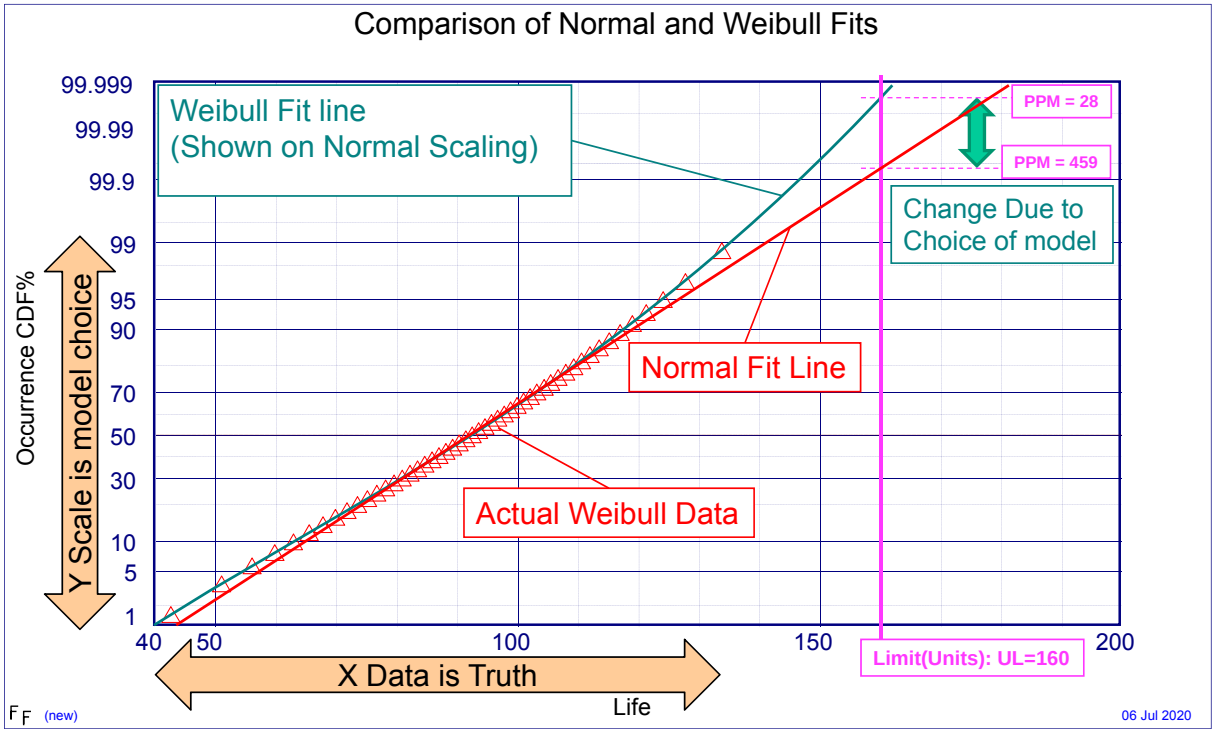
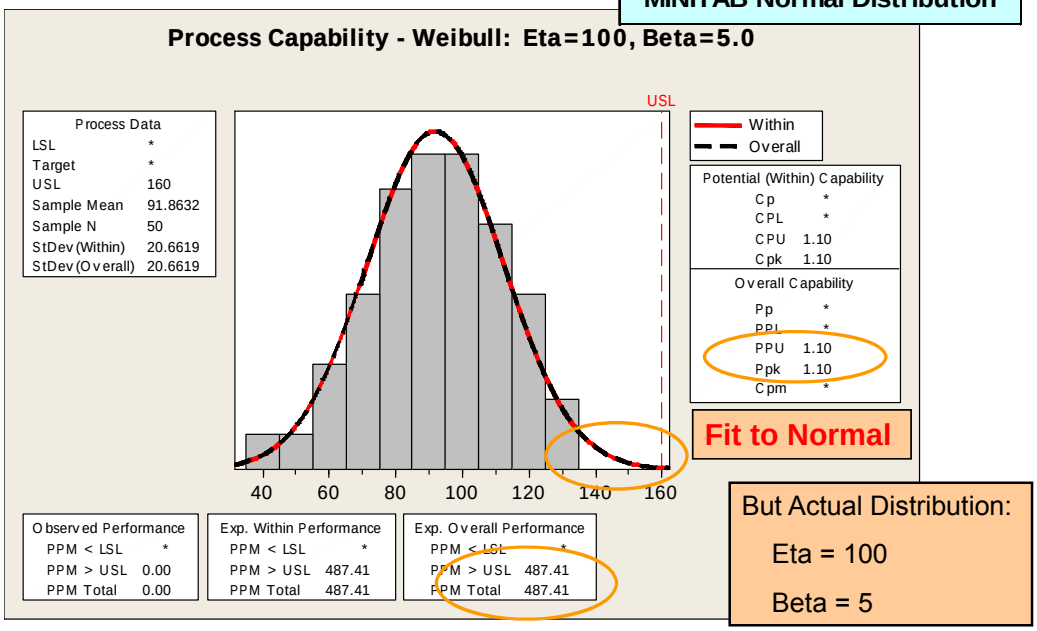


Weibull Data: Analyze with Normal Distribution

MINITAB Normal Distribution



Minitab has options to calculate StDev (e.g. rbar/d2, Sbar, Pooled SD). Be careful in comparisons to use the same method! (caution suggested by Carl Tarum)

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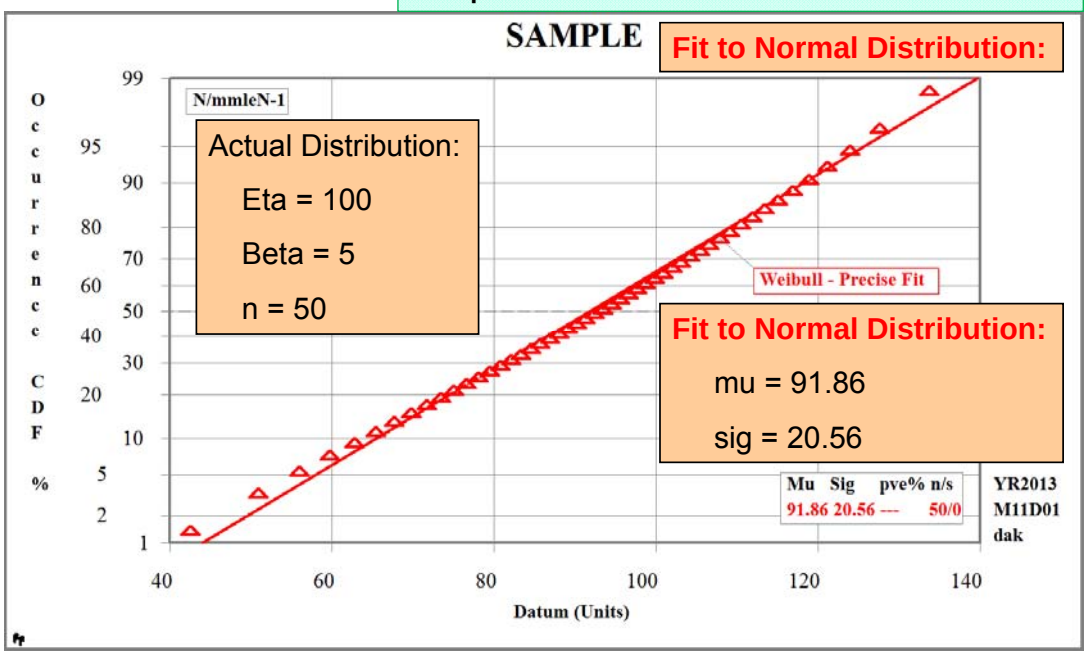
Weibull Data Analyzed with Normal Distribution

SuperSMITH Normal Distribution



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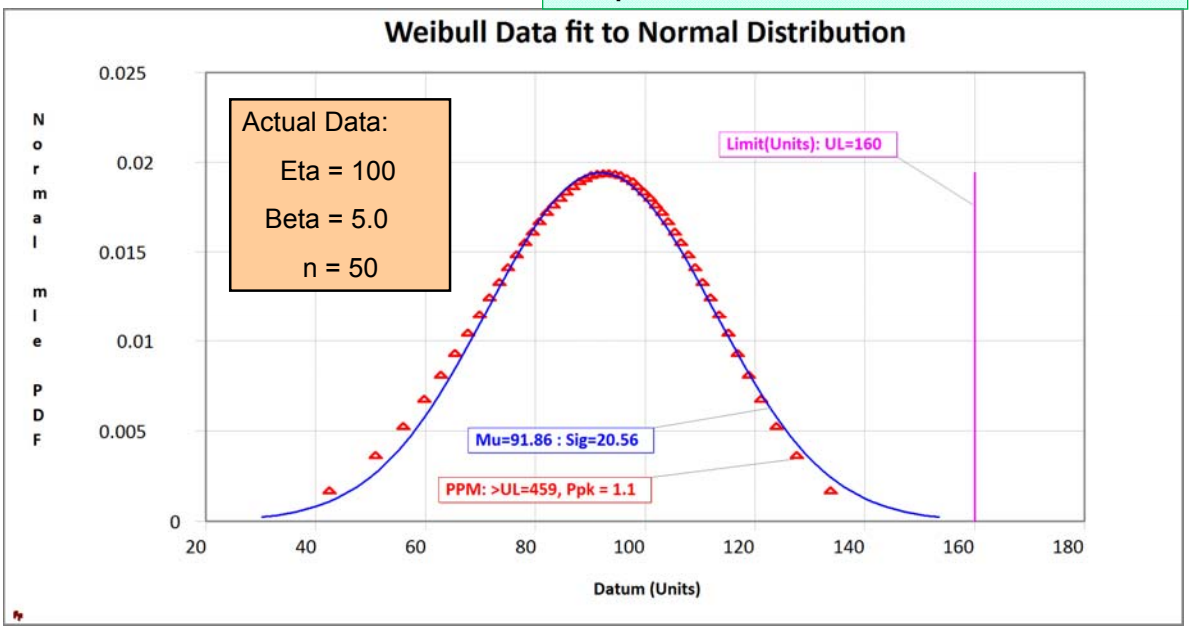
Weibull Data Analyzed with Normal Distribution

SuperSMITH Normal Distribution



Non-Normal Distribution

Dennis Keisic's PPM Method

Use the Actual PPM Value

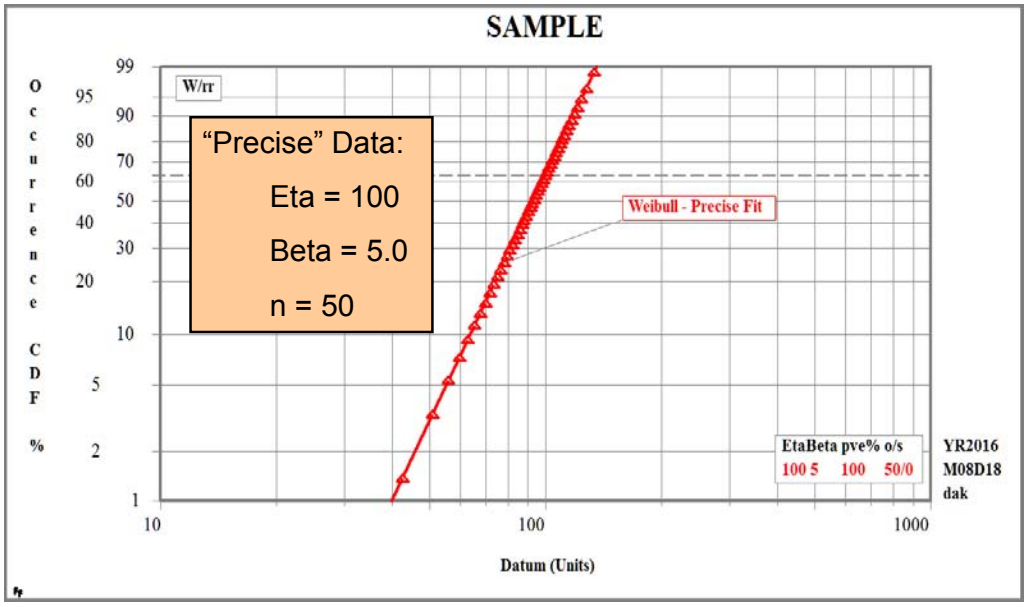
-then calculate the capability

- PPM
- Ppk
- Ppk with 1.5 sigma shift (or any shift)

* Double Limit
* Upper Limit
* Lower Limit

Non-Normal Distribution

PPM Method



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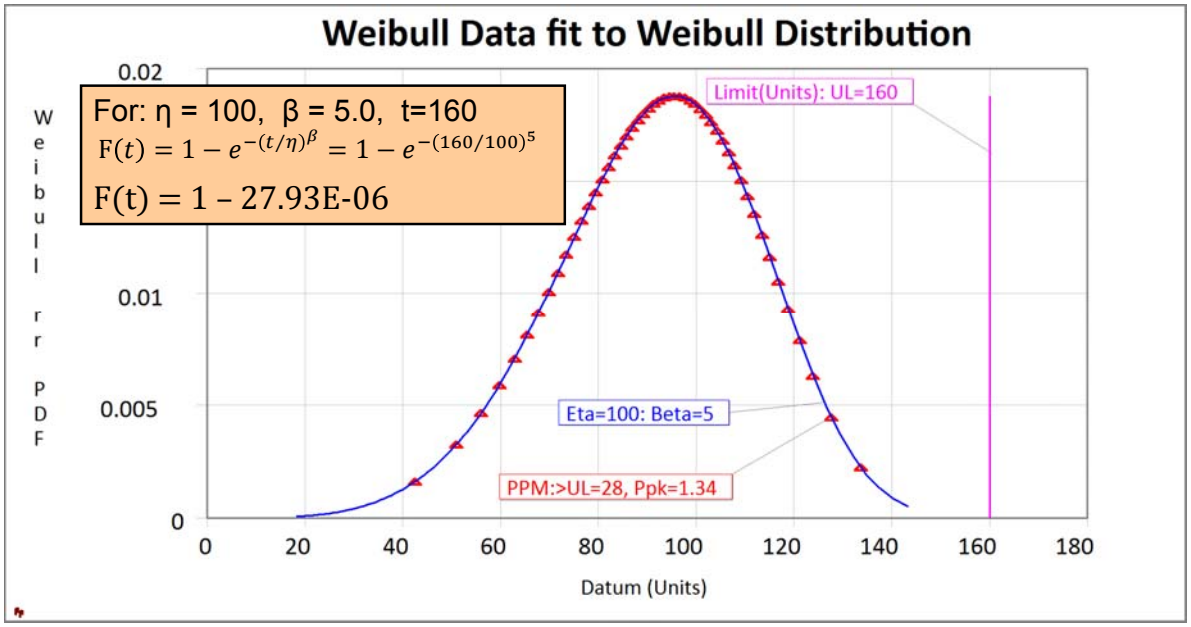
Non-Normal Distribution

PPM Method



SuperSMITH calculates the Ppk, by converting from PPM using standard z-Tables.

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Calculation of Ppu

Normal and Non-Normal Distributions

$$P_{pu} = \frac{USL - \mu}{3\sigma} = \frac{1}{3}\frac{USL - \mu}{\sigma} = \frac{z\text{-}score}{3}$$

For Normal Distribution:
μ = 91.86 , σ= 20.56

$$P_{pu} = \frac{1}{3}\frac{160 - 91.86}{20.56} = \frac{3.31}{3}$$

Z-Score = 3.31

$$P_{pu} = \frac{3.31}{3} = 1.10$$

For Weibull Distribution:
For: η = 100, β = 5.0, t=160

$$F(t) = 1 - e^{-(t/\eta)^{\beta}} = 1 - e^{-(160/100)^5}$$

F(t) = 1 – 27.93E-06

Z-Score = 4.03

$$P_{pu} = \frac{4.03}{3} = 1.34$$

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Non-Normal Distribution

PPM Method

Actual Data:
Eta = 100
Beta = 5.0
n = 50

Fit to Normal Distribution:
mu = 91.86
sig = 20.56
USL = 160
PPM = 459
Ppk = 1.10

Actual Weibull Distribution:
Eta = 100
Beta = 5.0
USL = 160
PPM = 28
Ppk (equivalent) = 1.34

"Free Lunch!"

Change: [(1.34 – 1.10) / 1.10] x 100

22 % increase in capability !!

Change PPM from 459 to 28 !!!

94 % decrease in PPM !!!

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Non-Normal Distribution

PPM Method

Normal Distribution (upper):

$$P_u = \frac{USL - mu}{3\ sigma}$$

Non-Normal (PPM Method)

Plot / Graph:
Weibull (2P, 3P), LogNormal, Exponential, Rayleigh, Gumbel, and even Normal!

$P_u = 1.0$ → PPM = 1350

"Still two parameter"

USL, Eta, Beta → PPM = 1350 → P_u (equiv)= 1.0

SuperSMITH calculates the Ppk, by converting from PPM using standard z-Tables.

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Assembly Plant - Capability Study - Cardan Joint

Case Study – JAT*

Serial #	(in-lbs)
7750	22.80
7857	17.10
7772	12.60
7749	19.50
7896	24.90
7760	28.50
7894	25.50
7745	14.10
7824	22.80
7893	31.80
7845	28.50
7751	16.80
7770	23.10
7755	22.20
7787	21.90
7791	18.90
7756	18.00
7831	15.30
7766	16.50

Serial #	
3360	21.00
3361	12.00
3351	29.40
3331	24.30
3349	24.30
3323	20.40
3359	24.00
3355	23.40
3350	34.80
3332	42.60
3358	28.80
3327	30.30
3353	39.00
3334	28.62
3346	36.00
3330	22.80
3343	26.40
3338	34.80
3290	31.20
3345	19.50
3317	36.00
3329	43.80
3294	21.90
3310	23.70
3352	46.50
3231	33.60
3298	16.80
3305	39.90
3295	18.00
3304	16.80
3315	23.40
3322	22.50
3342	23.10
3292	16.20
3357	26.40

3326	40.20
3311	27.60
3337	15.90
3319	45.30
3293	20.70
3307	19.80
3344	38.70
3316	30.60
3324	28.20
3318	33.00
3296	24.30
3335	20.40
3321	32.40
3336	18.60
3328	42.30
3288	38.70
3356	21.00
3299	27.00
3287	27.00
3312	32.40
3354	16.20
3320	34.50
3348	19.20
3300	33.90
3363	22.20
3308	32.70
3347	23.10
3340	32.10
3313	14.10
3286	25.50
3297	26.70
3325	18.60
3289	16.20
3339	28.50
3306	16.20
3303	24.00
3314	36.00
3291	22.20
3309	22.50
3285	30.00
3301	30.00
3302	26.70
3362	30.30
2896	40.50
2839	34.20

Sample size = 99

* JAT = Joint Articulation Torque (Single Cardan Universal Joints with Circlip retention)

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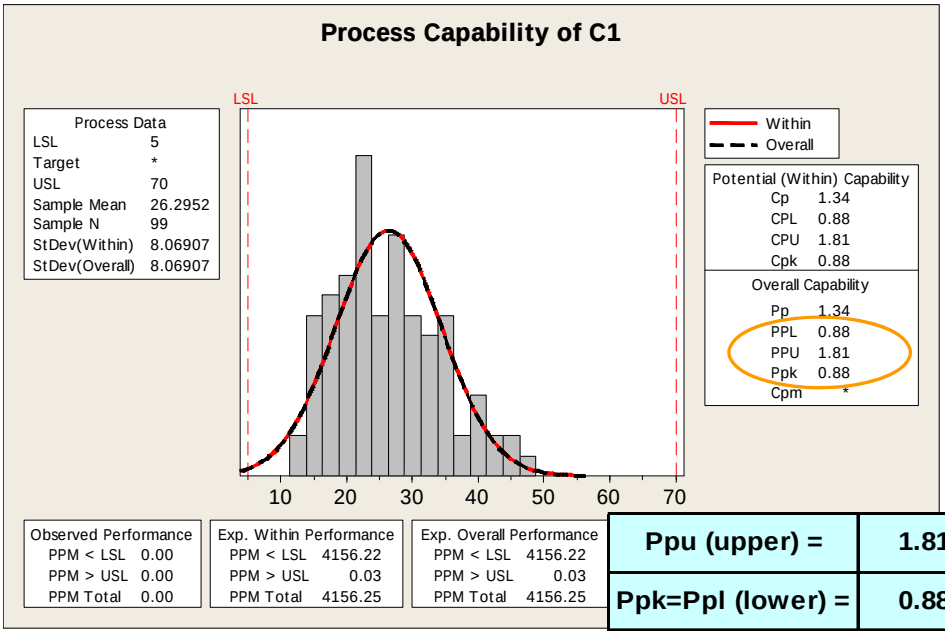
Case Study - JAT

MINITAB Normal Distribution



Ppu (upper) =	**1.81**
Ppk=Ppl (lower) =	**0.88**

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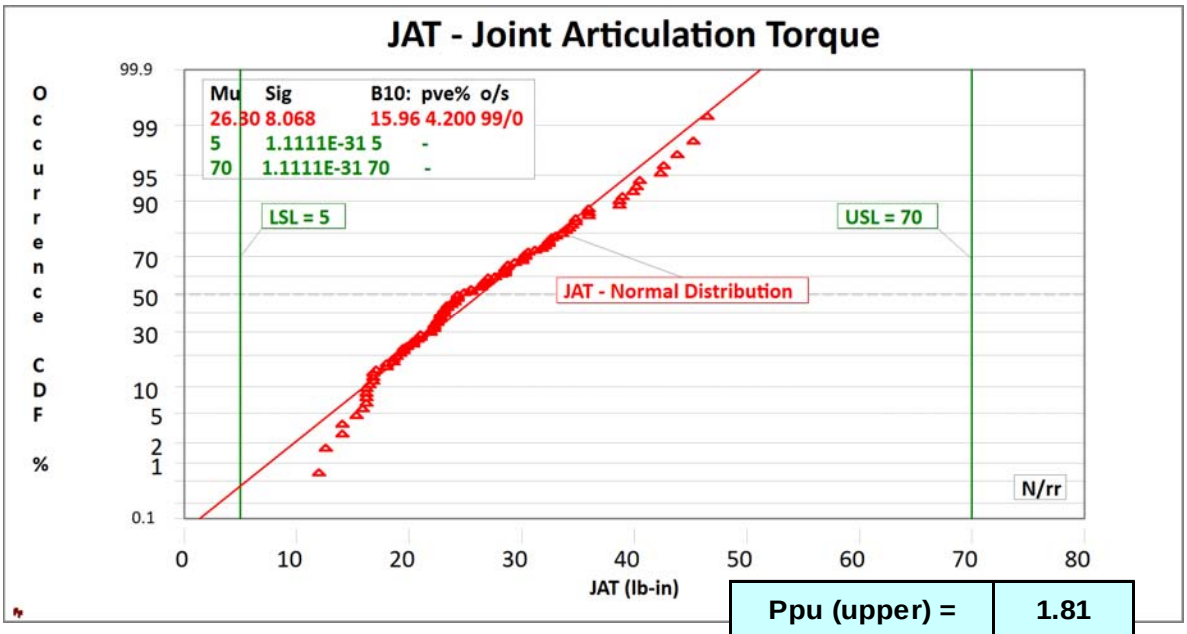
Case Study - JAT

SuperSMITH Normal Rank Regression



Ppu (upper) =	**1.81**
Ppk=Ppl (lower) =	**0.88**

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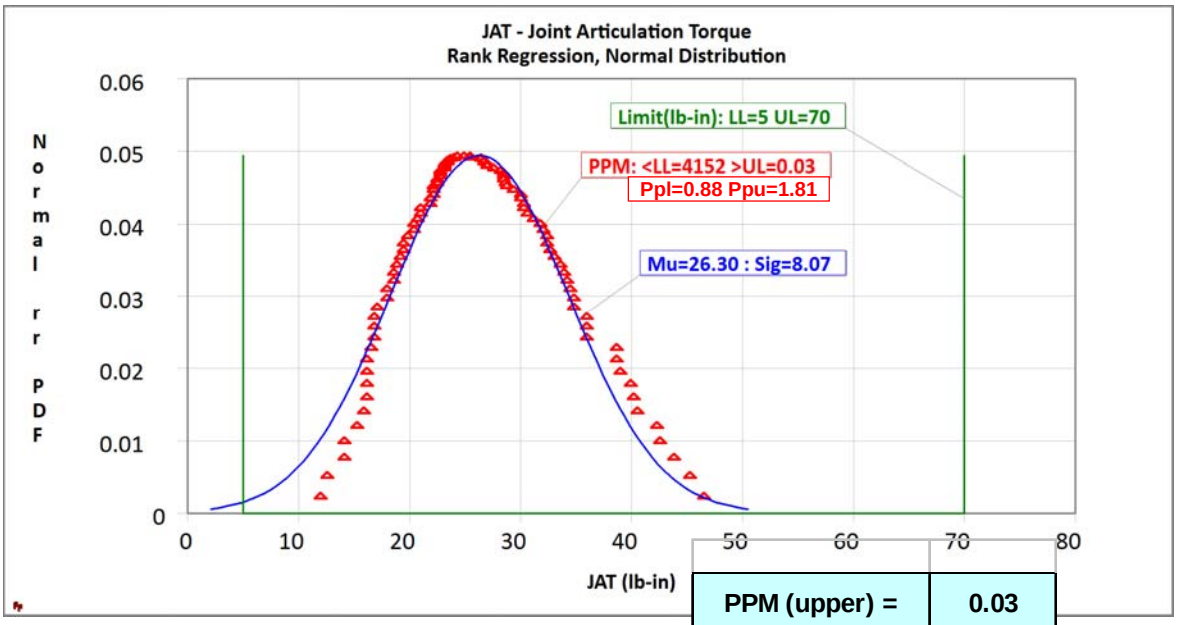
Case Study - JAT



PPM (upper) =	0.03
PPM (lower) =	4152

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Case Study - JAT

Life Data Distribution Analysis
Comparison Method: Regression Fit Value [pve%, P-value Estimate %]

Set1 - 'JAT - Weibull 3P'
W:Weibull [t0 = None ... 2 Parameter]
W: Corr.(r)=0.975 r^2=0.950 ccc^2=0.966 pve%=4.16 (Small)
W: Character. Value (Eta)=28.80 Weibull Slope (Beta)=4.124 Method=rr

3:Weibull [t0 = 10.37765 ... 3 Parameter] [Scale As Recorded]
3: Corr.(r)=0.996 r^2=0.993 ccc^2=0.983 pve%=74.25 (Big)
3: Character. Value (Eta)=28.35 Weibull Slope (Beta)=2.083 Method=rr/t0

L:LogNormal [t0 = None ... 2 Parameter]
L: Corr.(r)=0.995 r^2=0.991 ccc^2=0.976 pve%=73.49 (Big)
L: Log-Mean Antilog (Med)=25.10 Std. Dev. Factor (SigF)=1.368 Method=rr

N:Normal [t0 = None ... 2 Parameter]
N: Corr.(r)=0.986 r^2=0.972 ccc^2=0.978 pve%=4.2 (Small)
N: Mean (Mu)=26.30 Std. Deviation (Sig)=8.068 Method=rr

Optimum = Weibull [t0 = 10.37765 ... 3 Parameter] [Scale As Recorded]

For more detail on pve% see the SuperSMITH Help file or “The New Weibull Handbook” by Dr. Robert Abernethy

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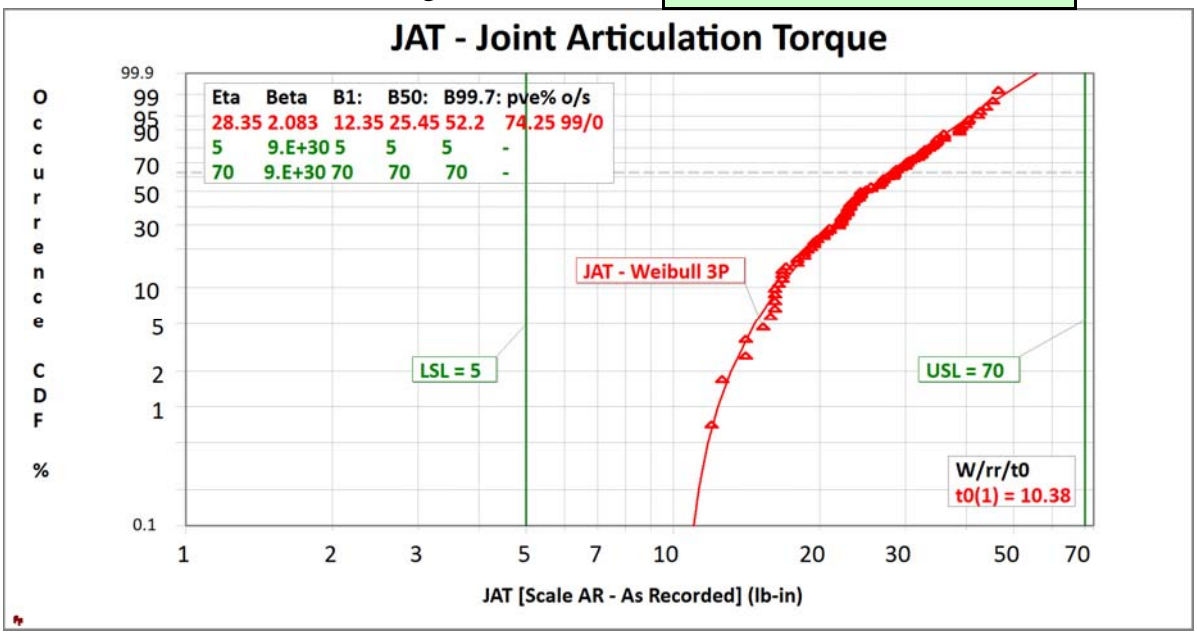
Case Study - JAT

Weibull 3 Parameter



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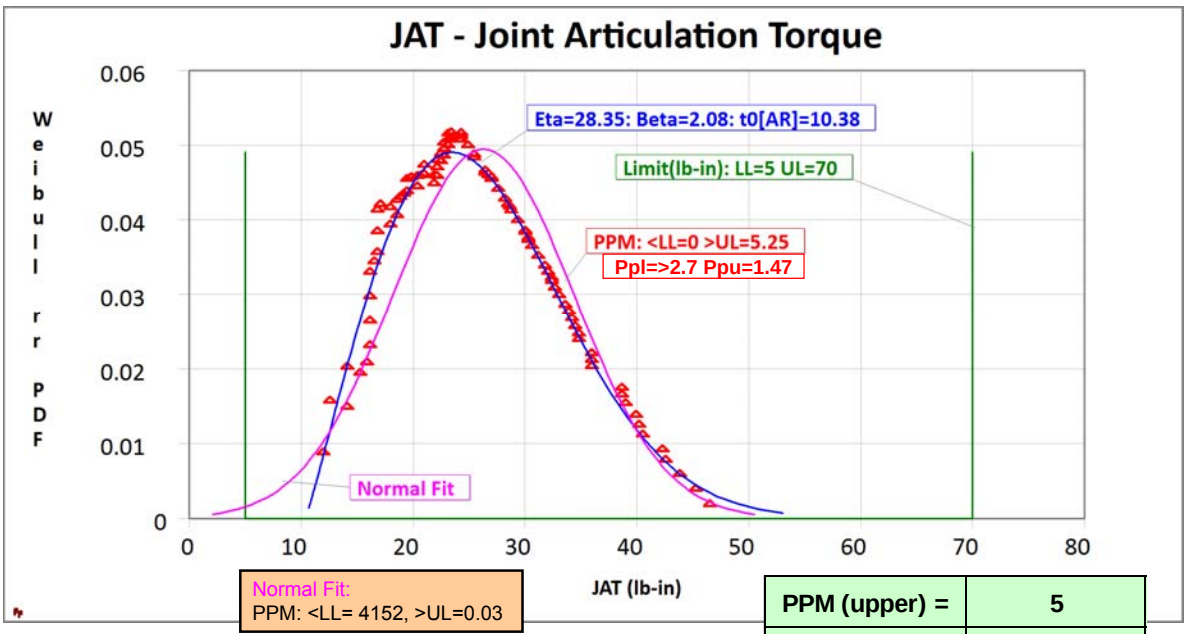
Case Study - JAT

Weibull 3 Parameter



Normal Fit:
PPM: <LL= 4152, >UL=0.03

PPM (upper) =	5
PPM (lower) =	0

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Case Study - JAT

Process Capability Index - Comparison "Short Term Capability (Zero shift)"				
"Free Lunch!"	Normal Rank Regression	Weibull Rank Regression	Weibull 3 Parameter	Log-Normal
PPM (upper) =	0.03	1	5	527
Sigma	5.00	4.76	4.60	3.27
P_p upper (equiv)	1.66	1.60	1.47	1.09
PPM (lower) =	4152	731	0	0.129
Sigma	2.60	3.20	6.00 +	5.16
P_p lower (equiv)	0.88	1.05	2.00 +	1.72

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Weibull and Six Sigma

"Free Lunch"

Story: The assembly specification for JAT was between 5 and 70 lb-in with Ppk>1.0. If no: 100% Inspection.

Reviewed data with Weibull 3 parameter, convinced customer that process was capable, and 100% inspection was not required.

As anything else, compromise.

Agreement was reached to "audit" 4 times per hour.

Everybody happy!

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Six Sigma Quality Limit Comparison (QLC)

Use the Actual PPM Value

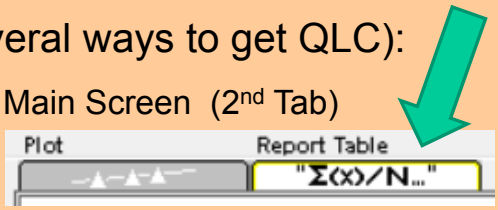
- then calculate the capability (equivalent)

SuperSMITH® Weibull (several ways to get QLC):

D: Options

H: Help+

Main Screen (2nd Tab)



- M: Plot / Report / Table
- V: View Additional Probability Plot with SSV …
- Q: QLC (Quality Limit Comparison)
  - PPM
  - Ppk
  - Ppk with 1.5 sigma shift (or any shift)

* Double Limit
* Upper Limit
* Lower Limit

SuperSMITH - Six Sigma Quality Analysis

PlayTIME™ Software Tutorial
Problem 47 – Plating Thickness

SuperSMITH – Non-Normal Example

PlayTIME™ Software Tutorial
Problem 54 – True Positioning

>>> Dennis's conversion chart is in the tutorial

>>> These examples and many others are available with the FREE DEMO software and FREE tutorial at **WeibullNEWS.com** … and … **FultonFindings.com** … and … **BathtubSoftware.com**

Software Tutorial
Problem 47

Quality Control Example Highlighting Advantages of Six Sigma with Weibull

From 2007 Gosset Challenge

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Electroplating Thickness
39 Measurements (cm)

0.0248	0.0285	0.0292	0.0301
0.0253	0.0285	0.0293	0.0301
0.0273	0.0285	0.0294	0.0302
0.0274	0.0286	0.0295	0.0303
0.0276	0.0288	0.0296	0.0304
0.028	0.0289	0.03	0.0308
0.0281	0.0291	0.03	0.0308
0.0283	0.0291	0.0301	0.0317
0.0283	0.0292	0.0301	0.032
0.0284	0.0292	0.0301	

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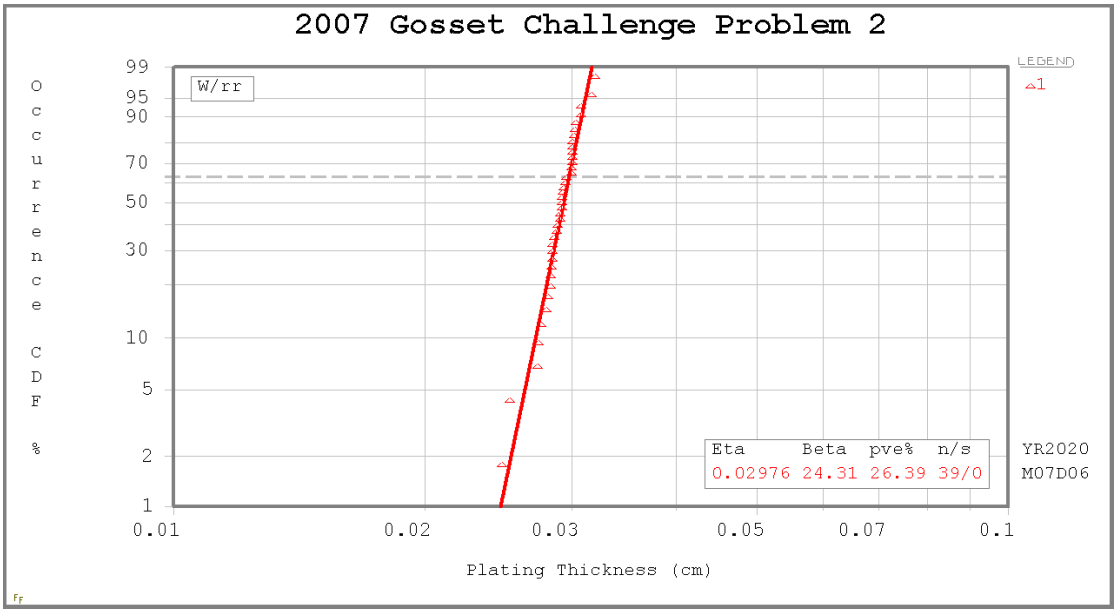
Weibull Plot



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Life Data Distribution Analysis
Note: Life Equations + Normal ... Select 'Method / Input' To Delete Normal
Comparison Method: Regression Fit Value [pve%]

Set1 - '1'
W:Weibull [t0 = None ... 2 Parameter]
W: Corr.(r)=0.979 r^2=0.959 ccc^2=0.936 **pve%=26.39** (Acceptable)
W: Character. Value (Eta)=.02976 Weibull Slope (Beta)=24.31 Method=rr
3:Weibull [t0 = -7.932429 ... 3 Parameter] [Scale As Recorded]
3: Corr.(r)=0.98 r^2=0.961 ccc^2=0.967 **pve%=5.12** (Poor)
3: Character. Value (Eta)=.02977 Weibull Slope (Beta)=6803.7 Method=rr/t0
L:LogNormal [t0 = None ... 2 Parameter]
L: Corr.(r)=0.955 r^2=0.912 ccc^2=0.952 **pve%=0.955** (Very Small)
L: Log-Mean Antilog (Med)=.02908 Std. Dev. Factor (SigF)=1.051 Method=rr
N:Normal [t0 = None ... 2 Parameter]
N: Corr.(r)=0.964 r^2=0.930 ccc^2=0.952 **pve%=2.44** (Small)
N: Mean (Mu)=.02912 Std. Dev. (Sig)=.0014400 Method=rr

Optimum = Weibull [t0 = None ... 2 Parameter]

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Select View Additional Probability Plot
Select “QLC” for Quality Limit Comparison

(1) Select “Double Limit”
- Enter 0.019 for Lower Limit
- Enter 0.036 for Upper Limit

(2) Confirm “PPM Type” = Yes

(3) Select “A…Activate…”, View plot

(4) Repeat 1 above, Set “Ppk Type” = Yes

(5) Select “A…Activate…”, View plot

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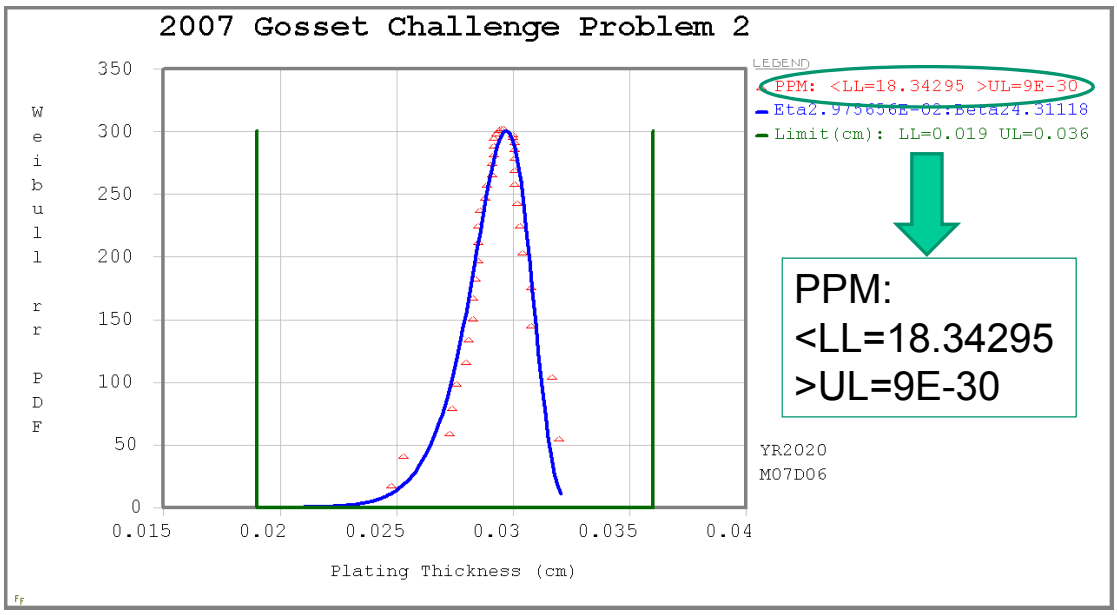
QLC Results in PPM



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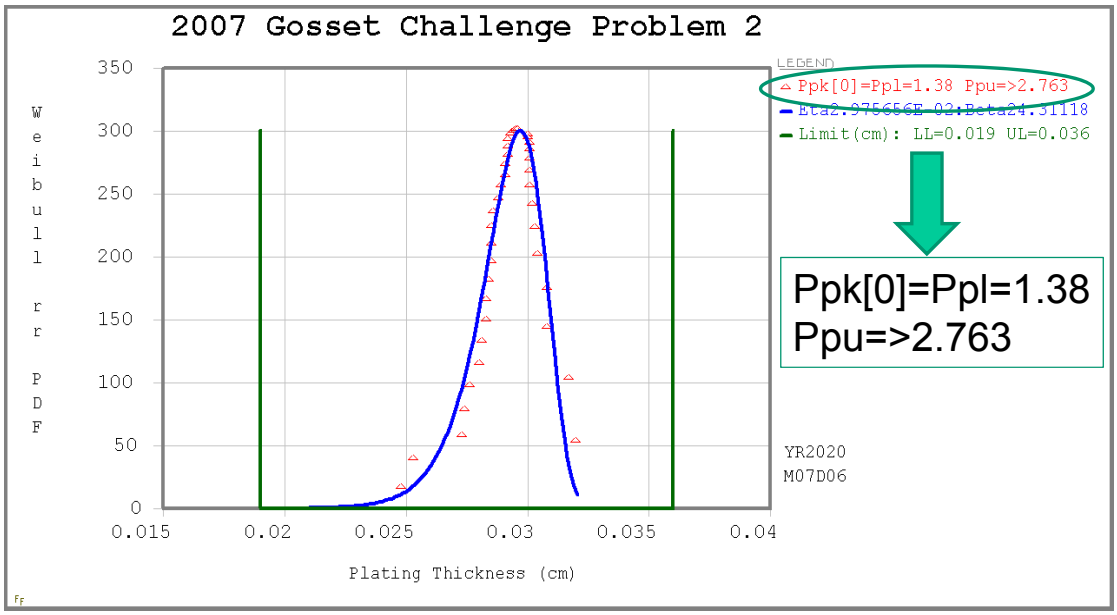
QLC Results in Ppk



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Six Sigma Work Is Not Limited to Normal

- Widen the applicability of programs and methods to all technical measurements
- Pick the most appropriate model rather than having to pick a poorer-fitting model
- Use Dennis Keisic's PPM equivalent recommendation … it gives:
  - PPM outside tolerance
  - Ppk indication (directly from PPM)
  - Cpk indication (with additional sequencing info)

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