Reliability Modeling for Dependent Competitive Failure Processes

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SUMMARY & CONCLUSION

In practical engineering applications, many factors of systems themselves and of random environments cause systems to suffer from degradation and shocks. Degradations, such as wear and erosion, occur in many systems, especially mechanical systems. Shocking is also a significant cause of system failure and hence has been paid more attention to. Shocks caused by the factors of the systems themselves usually have regular periods; especially for rotating devices, shocks have approximately fixed periods. Shocks caused by the factors of random environment usually follow Poisson process. In this paper, a new system reliability model is proposed for systems that involve dependent and competitive degradations and shocks. This model will have wide application in many fields. A numerical example is given to illustrate the model.

1 INTRODUCTION

There are three mainstream methods to deal with the deterioration of a system: (1) binary state theory, which only expresses two states: perfect functioning and complete failure; (2) continuous state theory, which is used to describe the continuous degradation process; and (3) multi-state system theory, which not only represents perfect functioning and complete failure, but also represents many medial states. Degradations exist in many systems because of the effect of systems and environments, such as wear, fatigue crack and erosion, and so on. The decrease of performance level is a gradual process. So far, there are many theoretical results and applications on the degradations of systems [1-3]. In ref.[1], binary system reliability is extended to continuous system reliability to resolve the problem of the continuous decrease of the system performance. The failure process is divided into two kinds: degradations and catastrophic processes. In order to obtain a mathematical degradation model statistical tool and regression are used. The relation between degradation and catastrophic failure is described by a state tree analysis and a fault tree analysis. In ref.[2], statistical degradation data are analyzed with two methods: the degradation path curve approach and the graphical approach. Similar results are obtained with these two methods. In order to overcome the disadvantages of the methods, a new method with a mixture model for continuous-state devices with catastrophic failure and degradation failure is proposed. To obtain enough failure data is time consuming in practice, therefore degradation data are used to assess the reliability of devices, especially of the highly reliable systems. Degradation is analyzed and a degradation model is constructed to evaluate the system reliability in ref.[3]. Bae [4] gave two kinds of degradation models, additive degradation model and multiplicative degradation model. In degradation reliability models, degradation data must be dealt with to fit a known distribution, maximum likelihood estimation is used in ref.[5]. Shocking is also a main factor to cause devices to fail, and more attentions have been focused on it recently. Mallor [6] classified shock models with independent assumption and a dependence structure respectively and defines a new general model. Based on an analysis of the cumulative shock model, the extreme shock model, and the δ-shock model, Bai et al. [7], formed a generalized framework of shock models. Li and Pham [8] analyzed a reliability model with two degradation processes and a random shock process. In this paper, the two degradation processes are decomposed into multiple states. Whichever process passes the prefixed critical value first would cause the system to fail. The system reliability evaluation can be solved with multi-state system theory. The organization of this paper is as follows. In section 2, the dependent competitive failure model (DCFDM) with degradation and shocks is described. A multi-state system reliability method to evaluate the reliability of DCFM is proposed in section 3. In section 4, a numerical example is used to illustrate the model. Some conclusions are given in section 5.

Notation

- $A_i$: Magnitude for the $i^{th}$ shock
- $A_{pc}$: The prefixed critical total magnitude of shocks
- $B_i$: Time interval between the $i^{th}$ and $(i+1)^{th}$ shock
- CDF: Cumulative distribution function
- C(δ): The prefixed critical region
- $D(t;X)$: Random degradation path
- $D_{uc}$: The prefixed critical degradation
- IFR: Increasing failure rate class
- $N(t)$: Poison distribution
- $r(t)$: Failure rate of degradation without considering the effect caused by shocks

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2 MODEL FORMULATION

A dependent competitive failure model is studied with the conditions:

1) The life distribution is subjected to IFR and the mean deterministic degradation path is defined as 
\[ \eta(t) = \alpha \exp(\beta t) \] 
2) Statistically, shocks follow a Poisson distribution \( N(t) \) and \( \{A_i, B_i\}_{i=1}^{\infty} \) is used to describe the whole shocking process, where \( A_i \) follows the normal distribution \( A_i \sim N(\mu_A, \sigma_A^2) \)
3) Shocks not only affect the performance level of the whole system directly, but also cause an indirect effect on degradation by a random variable \( Y_i \) for the \( i^{th} \) shock. And \( Y_i \) follows the normal distribution \( Y_i \sim N(\mu_Y, \sigma_Y^2) \)

The relation among degradation process, shocks and system performance is described in Figure 1.

![Figure 1 – The Relation among Degradation Process, Shocks and System Performance](image)

3 EVALUATE THE RELIABILITY WITH THE MULTI-STATE THEORY

Reliability analysis, especially for large systems, plays an important role in reliability engineering. It helps engineers to evaluate the current system performance and make maintenance decisions to reduce the probability of failure.

3.1 Shock Process Analysis

In engineering applications, many factors from the systems themselves and from the environment bring shocks to systems. Shocks caused by factors associated with the systems themselves usually have regular periods while shocks caused by random environmental factors usually follow Poisson processes.

Four principal shock models have been studied: 1) extreme shock model: a system fails as soon as the magnitude of any shock exceeds a prefixed critical level; 2) cumulative shock model: a system will break down when the cumulative magnitude of the shock exceeds the critical value; 3) run shock model: a system will work until \( k \) consecutive shocks with critical magnitude happen; and 4) \( \delta \)-shock model: a system will fail when the time lag between two successive shocks falls into \( C(\delta) \) ([6] and [7]). Here, the cumulative shock model is used with an equivalent reliability form:

\[ t \leq T \iff \sum_{i=1}^{N(t)} A_i \leq A_f \tag{1} \]

3.2 Degradation Analysis

Degradation data analysis is used to evaluate the reliability CDFM. Consider the group of degradation data in Table 1.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( y_{11} )</th>
<th>( y_{12} )</th>
<th>( \ldots )</th>
<th>( y_{1n} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_2 )</td>
<td>( y_{21} )</td>
<td>( y_{22} )</td>
<td>( \ldots )</td>
<td>( y_{2n} )</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \ddots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( t_n )</td>
<td>( y_{n1} )</td>
<td>( y_{n2} )</td>
<td>( \ldots )</td>
<td>( y_{nn} )</td>
</tr>
</tbody>
</table>

In Table 1, \( 1, 2, \ldots, n \), represent the serial number of the systems, from which the degradation data are sampled.

The mean degradation at \( t \) can be obtained by:

\[ \bar{y}_i = \frac{1}{n} \sum_{j=1}^{n} y_{ij} \tag{2} \]

A regression method is used to fit \( \bar{y}_i \), and \( t_i \) with a function \( \eta(t) \). Provided at any \( t_i \), the degradation process follows Weibull distribution, the degradation path can be expressed as:

\[ D(t; X) = \eta(t) + X \tag{3} \]

where \( X \) follows a Weibull distribution with the CDF:

\[ F(x) = 1 - \exp\left(-\left(\frac{x}{\beta}\right)^\alpha\right) \tag{4} \]

The CDF of the degradation process can be expressed as:

\[ p_i(\eta(t)+X \leq D_i) = p_i(X \leq D_i - \eta(t)) = \Phi\left(\frac{D_i - \eta(t)}{\beta}\right) \tag{5} \]

When the degradation exceeds \( D_i \), the system breaks down. Life distribution of the system can be written as:

\[ F(x) = \exp\left[-\left(\frac{x}{\beta}\right)^\alpha\right] \tag{6} \]

And the corresponding failure rate can be represented as:

\[ r(t) = -\frac{f(x)}{F(x)} = \eta(t) \exp\left[-\left(\frac{D_i - \eta(t)}{\beta}\right)^\alpha\right] \tag{7} \]

3.3 System Performance Analysis

Each shock causes the degradation process to change by a random variable. The continuous degradation process is divided into many different states with the changes caused by shocks. The state transmission chart under consideration with the effect process between degradation and shocks can be described as in Figure 2.

The \( i^{th} \) shock causes the system to divert from state \( i-1 \) to \( i \) (\( i = 1, 2, \ldots, n \)). The actual transition path is from state \( i-1 \) to state \( i \) without jumps as the solid line shown in Figure 2. In the process of calculation, the transition path considered is from state 0 to state \( j \) (\( j = 1, 2, 3, \ldots, n \)), described by the
The probability in state \( j \ (j = 0, 1, \ldots, n) \) can be expressed as:

\[
P_0 = p_r \{ N(t) = 0, \eta(t) + X < D_j, A_i < A_j \}
\]

\[
P_1 = p_r \{ N(t) = 1, \eta(t) + X < D_j, A_i < A_j \}
\]

\[
P_j = p_r \{ N(t) = j, \eta(t) + X + \sum_{i=1}^{j-1} Y_i < D_j, \sum_{i=1}^{j-1} A_i < A_j \}
\]

\[
P_n = p_r \{ N(t) = n, \eta(t) + X + \sum_{i=1}^{n} Y_i < D_j \}
\]

\[
F_{D_j}(t) = p_r \{ \eta(t) + X + \sum_{i=1}^{n} Y_i < D_j \}
\]

\[
= p_r \{ X + Y < D_j - \eta(t) \}
\]

If \( 1.5 \leq \alpha \leq 3.5 \), \( X \) is transformed to the normal distribution with small deviation by Eq. 17 and Eq. 18, respectively.

\[
\mu_X = \beta \Gamma[1 + \frac{1}{\alpha}]
\]

\[
\sigma_X = \beta^2 \Gamma[1 + \frac{2}{\alpha}] - \Gamma^2[1 + \frac{1}{\alpha}]\]

then \( X \) \( \sim \) \( N(\mu_X, \sigma_X) \)

\( A_i \ (i = 1, 2, \ldots, n) \) are independent and follow normal distribution with the CDF:

\[
F(a_i) = \int_{-\infty}^{a_i} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt
\]

Thus, \( A = \sum_{i=1}^{n} A_i \) also follows a normal distribution with the CDF:

\[
F_A(a) = \int_{-\infty}^{a} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt
\]

The system reliability is calculated by:

\[
R = \sum_{j=0}^{n} P_j
\]

To obtain the system reliability, the probability that the system is in every state must be known. \( Y_i \ (i = 1, 2, \ldots, n) \) are statically independent and follow normal distribution with the CDF:

\[
F(y_i) = \int_{-\infty}^{y_i} e^{-\frac{(t-\mu_i)^2}{2\sigma_i^2}} dt
\]

According to the additive characteristic of the normal distribution, \( Y = \sum_{i=1}^{n} Y_i \) also follows a normal distribution with the CDF:

\[
F(y) = \int_{-\infty}^{y} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt
\]

The degradation process under the \( i^{th} \) shock can be written as:

\[
D(t) = \eta(t) + X + \sum_{i=1}^{n} Y_i
\]

with the CDF:

4 NUMERICAL EXAMPLES

Let \( X \) follows the Weibull distribution with parameters \( \alpha = 2, \beta = 1.2 \); \( Y_i \) follows normal distribution with mean \( \mu_i = 6 \) and variance \( \sigma_i^2 = 2 \); \( A_i \) follows normal distribution with mean \( \mu_i = 4 \) and variance \( \sigma_i^2 = 2 \); the prefixed
degradation critical value is $D_f = 800$ and shocks is critical $A_f = 600$; the parameter $a = 0.7, m = 0.7$ in $\eta(t) = a \exp(mt)$ and the parameter $\lambda = 0.4$. When the system is in state $n$, the $n^{th}$ state probability can be calculated as follows. Firstly, we transform the Weibull distribution to normal distribution.

$$\mu_X = \beta \frac{[1 + \frac{1}{\alpha}]}{\alpha} = 1.2 \frac{\Gamma[1 + \frac{1}{2}]}{\sqrt{\pi}} = 0.6 \sqrt{\pi}$$

$$\sigma_X = \beta \frac{2}{\alpha} \Gamma[1 + \frac{1}{\alpha}] - \Gamma^2[1 + \frac{1}{\alpha}] = 1.2^2 \frac{(1 + \frac{2}{\alpha})}{\sqrt{\pi}}$$

When the system is in state $n$, the $n^{th}$ state probability can be calculated as follows. Firstly, we transform the Weibull distribution to normal distribution.

$$X \sim \text{Weibull}(\mu, \sigma)$$

$$F_X(x) = \frac{1}{\sqrt{2\pi} \sigma} \int_{-\infty}^{x} \frac{1}{\sigma} e^{-\frac{(x - \mu)^2}{2\sigma^2}} dx$$

Then $Z = X + Y$ follows a normal distribution with the CDF:

$$F_Z(z) = \frac{1}{\sqrt{2\pi} \sigma} \int_{-\infty}^{z} \frac{1}{\sigma} e^{-\frac{(z - \mu)^2}{2\sigma^2}} + 2n) dx$$

We obtain the CDF under the consideration of the effect of shocks.

$$F_B(t) = p \eta(t) + X + \sum_{i=1}^{n} Y_i < D_f$$

$$= p \{ X + Y < D_f - \eta(t) \}$$

$$= F_Z(800 - 0.7 e^{0.7i})$$

The system probability in state 0 and $n$ can be expressed as:

$$P_0 = p, \{ N(t) = 0, \eta(t) + X < D_f \}$$

$$= p, \{ N(t) = 0 \} p, \eta(t) + X < D_f$$

$$= e^{-0.4t} * F_Z(800 - 0.7 e^{0.7i})$$

$$P_n = p, \{ N(t) = n, \eta(t) + X + \sum_{i=1}^{n} Y_i = D_f, \sum_{i=1}^{n} A_i < A_f \}$$

$$= p, \{ N(t) = n \} p, \eta(t) + X + \sum_{i=1}^{n} Y_i < D_f, \sum_{i=1}^{n} A_i < A_f$$

$$= e^{-0.4t} * F_Z(800 - 0.7 e^{0.7i}) * \sum_{i=1}^{n} \frac{(4t)^i}{i!} e^{-0.4t} * F_Z(800 - 0.7 e^{0.7i}) * \frac{1}{2 \sqrt{2\pi i}} e^{-\frac{(i-1)^2}{8i}} dt$$

The probability of state 0, state 8, and the system reliability are depicted in Figures 3, 4, and 5, respectively.

![Figure 3 Probability in State 0](image)

![Figure 4 Probability in State 8](image)

![Figure 5 System Reliability](image)

In Figures 3 and 4, we know that the probability decreases when $n$ increases. When the system reaches the 8th state, the probability is approximately 0.02. In Figure 5, we see that the system reliability remains stable.

### 5 CONCLUSION

In this paper, a new model to deal with dependent competitive failure processes is proposed. Shocks not only bring a random direct effect to the system, but also bring degradation, a random indirect effect. As the random indirect effect that causes the degradation cannot be expressed by a uniform function, and multi-state system reliability theory is employed here. The difference from traditional multi-state system theory is that the jumping time from state $i$ to $i+1$ is a random variable. This causes every state probability to be a function of $t$. This model will have wide application in many...
fields. For example, diesel engines are subject to such degradation processes as wear, erosion, and so on; shocks are also caused by random environmental not only on the diesel engines themselves but also on the degradation processes of the diesel engines, especially in the case that the vibration is very significant in specific frequency ranges. This model can be used to estimate the reliability of diesel engines. The method also provides engineers with a tool for design optimization and maintenance decision-making. More theoretical and application research will be conducted.

REFERENCES

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