Improved Method for ALT Plan Optimization - Revisited

Peter Arrowsmith
BOTE Consulting
Outline

• ALT planning & compromises
• Extrapolated value of interest, B10 life at use
• Prediction objective
• ALT terminology
• Simulation method & results
• Prediction model for test plan optimization
• Model validation
• Conclusions
• Backup slides: alternative methods, model statistics
ALT Planning Overview

• The test plan must be optimized in terms of appropriate stresses, the number of units under test ($n_t$) and allocation among the stress levels.
• Practical considerations include:
  • Cost & time budgets, limited $n_t$ and test duration or censor time ($\eta_c$)
  • Sufficient number of failures to give statistically valid results
  • Too high stress levels, spurious failure modes
• Various approaches to optimize ALTs have been investigated. This work builds on the paper by Ma & Meeker (M&M):
3-Stress Level, Weibull-Arrhenius, Life-Stress Model

--- extrapolation to 10% failure at $T_{USE} = 50 \, ^{\circ}C$
Prediction Objective

• Develop method for one step ALT optimization (Excel)
• Minimize the variance of the extrapolated time to failure
• Focus on small sample sizes $n_t$ 20-60 units
• Obtain candidate test plans with stress levels and sample allocations, within ±2 units of the optimal
  • If needed, candidate plans can be fine-tuned using Monte Carlo simulation
• Single constraint of equally spaced stress levels, on standardized scale
• Inputs are the test plan estimated values, life model, number of test units and probability of zero failures:
  • $\Pr\{\text{ZFP1}\}$ is a useful metric for comparing ALT plans
Why Another Test Plan Optimization Method?

• For a 3 level, compromise test plan with stress levels: low (L), middle (M) & high (H)

• The commonly used fixed allocation of units, e.g. $n_L:n_M:n_H = 4:2:1$ or fractional allocation $n_M/n_t = \pi_M = 0.2$, may not be optimal

• Is fixed allocation appropriate for a wide range of sample size $n_t$?

• What if >3 stress levels?

Note: although a minimum of only 2 levels are required for single stress (3 unknown parameters), compromise test plans with 3 levels have several advantages.
ALT Terminology (1)

• Assume a log time-to-failure (TTF) distribution with constant scale \( \sigma \), with CDF:  
  \[ F(t) = \Phi[(\ln(t) - \mu)/\sigma] \]

• The location \( \mu \) has a linear dependence on the standardized stress \( \xi \) (\( Xi \)):  
  \[ \mu = \gamma_0 + \gamma_1 \cdot \xi \]
  • \( \sigma, \gamma_0 \) & \( \gamma_1 \) are determined by fit to the test data
  • based on the lowest or use (U) and highest (H) stress, the standardized stress (0-1):  
    \[ \xi = (s-s_U)/(s_H-s_U) \]
    where \( s \) is the transformed stress, e.g. \( 1/T_{abs} \)

• Estimated, best-guess input values needed for test planning:
  • life probability distribution and scale factor \( \sigma \) (1/\( \beta \), Weibull)
  • failure probabilities at 2 conditions, e.g. \( pf_U \), \( pf_H \), and time \( \eta_c \)
  • Or, one probability, time and a stress parameter, e.g. activation energy \( E_a \) for Arrhenius, exponent for power law
ALT Terminology (2)

• Can solve for $\gamma_0$ & $\gamma_1$ and estimate the probability of failure $pf_i$ of a unit at any stress level $i$

• Assume a compromise test plan with 3 levels: L, M, H

• To reduce the unknowns, impose the constraint of equally spaced (standardized) stress levels $\xi_M = (\xi_L + \xi_H)/2$

• The allocation of the units is: $n_t = n_L + n_M + n_H$
  • or fractional allocation $1 = \pi_L + \pi_M + \pi_H$, $\pi_L = n_L/n_t$

• The metric of interest is the log-time ($y_p$) for the $p^{th}$ percentile failure (e.g. $p=10\%$) at the use condition, the goal of the optimization is to minimize the scaled variance: $(n/\sigma^2).\text{Var}(y_p)$

• M&M utilized $\text{Pr}\{\text{ZFP1}\}$, the probability of zero failures at one or more stress levels. $\text{Pr}\{\text{ZFP1}\}$ depends on the failure $pf_i$
  • $\text{Pr}\{\text{ZFP1}\} = 1-(1-R_L).(1-R_M).(1-R_H)$, where $R_L = (1-pf_L)^n_L$
Summary of Key Parameters

• Three stress levels L, M & H; allocations: n_L, n_M & n_H
• Lowest stress ξ_L, on a standardized scale, 0-1
  • constraint of equally spaced levels... always applied
• Probability of zero failures, typical test plan values with Pr{ZFP1} = 1% or 5%
• Result of interest: the estimated time to 10% failure, extrapolated to the use condition, t_{p=0.1}
• Goal is to minimize the error in the estimate, or the scaled value: (n/σ^2).Var[ln(t_{p=0.1})]
  • Shorthand: (n/σ^2).Var(y_{p=0.1}) or "Var"
Optimization Method

• The lower stress level $\xi_L$, allocation $n_L$ and $\text{Pr}\{\text{ZFP1}\}$, are interdependent:
  • for a given $n_L$: as $\xi_L \uparrow$ $\text{Pr}\{\text{ZFP1}\} \downarrow$
  • for given $\text{Pr}\{\text{ZFP1}\}$: as $n_L \downarrow$ $\xi_L \uparrow$

• For a given allocation, the smaller $\xi_L$ or the wider the stress level spacing, the smaller the expected variance of the TTF, extrapolated to the use condition

• This suggests the optimization approach; for each combination of $n_L$ and $n_M$ (or $n_H$) find the minimum $\xi_L$ that achieves the target $\text{Pr}\{\text{ZFP1}\}$

• In Excel make an $n_L \times n_H$ lookup table of calculated minimum $\xi_L$ values (see over)
## Candidate Test Plan Selection

### Weibull life with 1 stress factor, Arrhenius (T) or Inverse Power Law model

**Input Test Planning values:**
- **Arrhenius or IPL? (A/T):**
  - 0.7950
- **T (°C):**
  - 25.0
  - 55.0
- **n_L:**
  - 100
- **Pr(Failure):**
  - 0.0000
  - 0.0000
  - 0.0000
  - 0.0000
  - 0.0000
- **ξ:**
  - 1.0088
- **Inverse stress Coeffs:**
  - 9.7580
  - 1.0000
  - 1.0000
  - 9.7560

### Standardized Stress matrix

<table>
<thead>
<tr>
<th>T (°C)</th>
<th>n_L</th>
<th>Pr(Failure)</th>
<th>ξ</th>
<th>Inverse stress Coeffs</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.0</td>
<td>100</td>
<td>0.0000</td>
<td>9.7580</td>
<td>1.0000</td>
</tr>
<tr>
<td>55.0</td>
<td>100</td>
<td>0.0000</td>
<td>9.7560</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

### Output stress levels & allocation for ξ_L=0.7246, n=40, min(n_M)=3

- **Allocation for given n that minimizes ξ_L for given Pr(ZFP1) target**
- **n_L:**
  - 100
  - 117
  - 20% (target Pr(Fail), to estimate censor time at each level)
- **n_M:**
  - 100
  - 84
  - 40%
- **n_H:**
  - 100
  - 72
  - 20%
- **Min ξ_L in lookup table:**
  - 0.6718
  - T = 82.8°C
- **Min ξ_L for est. alloc. table:**
  - 0.7246

### Est. allocation (L & M models)

<table>
<thead>
<tr>
<th>L</th>
<th>M</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>0.6250</td>
<td>5.7</td>
</tr>
<tr>
<td>6.5475</td>
<td>3.3</td>
<td></td>
</tr>
<tr>
<td>0.9000</td>
<td>5.0</td>
<td></td>
</tr>
<tr>
<td>10.0000</td>
<td>12.0000</td>
<td></td>
</tr>
</tbody>
</table>

### Check Pr(ZFP1)

- 0.0100

### Table of minimum ξ_L values

<table>
<thead>
<tr>
<th>n_L / n_H</th>
<th>ξ_L</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.7843</td>
</tr>
<tr>
<td>4</td>
<td>0.7917</td>
</tr>
<tr>
<td>5</td>
<td>0.7973</td>
</tr>
</tbody>
</table>

**n_L** by **n_H** table of minimum ξ_L for Pr(ZFP1)=1%

![ASQ Reliability & Risk Division](image)
Evaluation of Candidate Test Plans

• Investigate the dependence of the scaled variance \((n/\sigma^2).\text{Var}(y_{p=0.1})\), on allocation and \(\xi_L\)

• Methodology:
  • For given \(n_t\), find minimum \(\xi_L (<0.95, \text{step-size 0.0001})\) for each \(n_L, n_M \& n_H (\geq 3)\) allocation, that meets the target \(\text{Pr}\{ZFP1\}\)
  • Estimate \text{Var} at the Use condition by Monte Carlo simulation*
  • 10,000 MC trials for each test plan, mean of 3 Var values
  • Plot \text{Var} vs. \(\xi_L\) and label test plans by allocation

*R programs, using functions from Bill Meeker’s RSPLIDA package: http://www.public.iastate.edu/~stat533/
Test Plan Comparison; \( n_t = 15 \)

• Allocations are labelled: \( n_L-n_M-n_H \). Constrained is equal failures at L & M.

• For a given allocation, the smaller \( \xi_L \) and more widely spaced stress levels, correspond to smaller variance.

• Small differences of \( \Delta \xi_L < 0.007 \) are negligible (1 °C) Monte Carlo error is 10 units (1 s.d. at Var \( \approx 400 \)).

Planning values:
\( n=15 \)
\( p_{f_L}=0.001, \ p_{f_H}=0.9 \)
\( \sigma=0.6 \)
\( \eta_c = \eta_L = \eta_M = \eta_H \)
\( \Pr\{ZFP1\} \leq 1\% \) (varies with \( \xi_L \))
Effect of Sample Size & Stress Level

• For small total n < 25, scaled variance increases as order $1/n^2$, more rapidly than the expected $1/\sqrt{n}$

• Additional source of error arises from stress level spacing; width of the stress levels is 50% smaller for $n=12$ ($\xi_L = 0.89$), compared to $n=30$ ($\xi_L = 0.75$)

• $\xi_L$ can be lowered and Var reduced by raising $Pr\{ZFP1\}$

Planning values:
$pf_U=0.001$, $pf_H=0.9$
$\sigma=0.6$
$\eta_c = \eta_L = \eta_M = \eta_H$
$Pr\{ZFP1\}=1%$
What is the Optimum Allocation?

- Investigate all practical allocation permutations for given $n$ and target $\text{Pr}\{\text{ZFP1}\}$
- Each allocation corresponds to the minimum $\xi_L$

Planning values:
- $n=40$
- $p_f^U=0.001$, $p_f^H=0.9$
- $\sigma=0.6$
- $\eta_c = \eta_L = \eta_M = \eta_H$
- $\text{Pr}\{\text{ZFP1}\}=1\%$

Each data point is a different allocation.
Note the minimum at $n_H$ 10 to 12
Var Response Surface

- Each allocation is based on the minimum $\xi_L$

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>n / n_r &gt;&gt;</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>629.7</td>
<td>534.8</td>
<td>473.1</td>
<td>432.1</td>
<td>400.0</td>
</tr>
<tr>
<td>10</td>
<td>541.4</td>
<td>455.1</td>
<td>406.8</td>
<td>372.1</td>
<td>348.4</td>
</tr>
<tr>
<td>11</td>
<td>471.7</td>
<td>403.0</td>
<td>360.9</td>
<td>328.4</td>
<td>306.4</td>
</tr>
<tr>
<td>12</td>
<td>422.3</td>
<td>360.3</td>
<td>321.7</td>
<td>296.0</td>
<td>277.8</td>
</tr>
<tr>
<td>13</td>
<td>384.1</td>
<td>325.2</td>
<td>295.5</td>
<td>270.2</td>
<td>253.4</td>
</tr>
<tr>
<td>14</td>
<td>353.0</td>
<td>304.4</td>
<td>271.6</td>
<td>250.0</td>
<td>234.9</td>
</tr>
<tr>
<td>15</td>
<td>325.8</td>
<td>282.4</td>
<td>254.7</td>
<td>234.6</td>
<td>219.8</td>
</tr>
<tr>
<td>16</td>
<td>306.7</td>
<td>265.8</td>
<td>238.5</td>
<td>222.2</td>
<td>208.4</td>
</tr>
<tr>
<td>17</td>
<td>288.6</td>
<td>251.5</td>
<td>227.0</td>
<td>209.6</td>
<td>198.0</td>
</tr>
<tr>
<td>18</td>
<td>274.9</td>
<td>239.2</td>
<td>216.2</td>
<td>201.3</td>
<td>190.4</td>
</tr>
<tr>
<td>19</td>
<td>261.7</td>
<td>230.5</td>
<td>208.3</td>
<td>195.3</td>
<td>183.4</td>
</tr>
<tr>
<td>20</td>
<td>253.1</td>
<td>219.9</td>
<td>203.1</td>
<td>188.6</td>
<td>178.3</td>
</tr>
<tr>
<td>21</td>
<td>245.2</td>
<td>213.9</td>
<td>196.1</td>
<td>183.5</td>
<td>174.0</td>
</tr>
<tr>
<td>22</td>
<td>237.1</td>
<td>207.2</td>
<td>190.6</td>
<td>180.4</td>
<td>171.6</td>
</tr>
<tr>
<td>23</td>
<td>230.9</td>
<td>204.0</td>
<td>187.1</td>
<td>174.0</td>
<td>168.7</td>
</tr>
<tr>
<td>24</td>
<td>225.3</td>
<td>200.5</td>
<td>183.7</td>
<td>173.5</td>
<td>166.3</td>
</tr>
<tr>
<td>25</td>
<td>225.4</td>
<td>197.2</td>
<td>185.0</td>
<td>172.0</td>
<td>164.0</td>
</tr>
<tr>
<td>26</td>
<td>219.4</td>
<td>194.5</td>
<td>180.7</td>
<td>170.8</td>
<td>164.3</td>
</tr>
<tr>
<td>27</td>
<td>216.4</td>
<td>193.6</td>
<td>178.0</td>
<td>168.7</td>
<td>165.5</td>
</tr>
<tr>
<td>28</td>
<td>212.3</td>
<td>193.7</td>
<td>177.9</td>
<td>172.7</td>
<td>181.0</td>
</tr>
<tr>
<td>29</td>
<td>214.9</td>
<td>192.4</td>
<td>182.3</td>
<td>190.7</td>
<td>254.0</td>
</tr>
<tr>
<td>30</td>
<td>213.8</td>
<td>197.2</td>
<td>205.1</td>
<td>272.1</td>
<td>660.9</td>
</tr>
</tbody>
</table>

Planning values:
- $n=40$
- $p_{f_U}=0.001$, $p_{f_H}=0.9$
- $\sigma=0.6$
- $\eta_c = \eta_L = \eta_M = \eta_H$
- $Pr\{ZFP1\}=1\%$
Over-allocation of L & H Levels is Generally Optimal for $n_t > 30$

<table>
<thead>
<tr>
<th>Test Plan values</th>
<th>Allocation</th>
<th>Expected Failures</th>
<th>$\xi_L$</th>
<th>$\Pr{ZFP1}$</th>
<th>Var</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n_L$</td>
<td>$n_M$</td>
<td>$n_H$</td>
<td>$nf_L$</td>
<td>$nf_M$</td>
</tr>
<tr>
<td>$n=40$, $p_U=0.001$, $p_H=0.9$, $\sigma=0.6$, $\eta_L=183$, $\eta_M=183$, $\eta_H=183$</td>
<td>25</td>
<td>4</td>
<td>11</td>
<td>5.5</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>23</td>
<td>6</td>
<td>11</td>
<td>5.6</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>21</td>
<td>8</td>
<td>11</td>
<td>5.9</td>
<td>4.6</td>
</tr>
<tr>
<td></td>
<td>22</td>
<td>8</td>
<td>10</td>
<td>6.0</td>
<td>4.6</td>
</tr>
<tr>
<td>$n=30$, $p_U=0.001$, $p_H=0.9$, $\sigma=0.6$, $\eta_L=183$, $\eta_M=183$, $\eta_H=183$</td>
<td>16</td>
<td>6</td>
<td>8</td>
<td>4.5</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>6</td>
<td>7</td>
<td>4.7</td>
<td>3.5</td>
</tr>
</tbody>
</table>

- Optimal over-allocates L, to obtain lowest $\xi_L$ for the target $\Pr\{ZFP1\}$
- Also, re-allocates from M to H
- "Pinning" effect; reduces the variance of the extrapolation
- Optimal allocations are typically not 4:2:1, $\pi_M = 0.2$ or equal L & M failures
Investigation of Optimal Plans

• For a range of input parameters, total of 95-2 test plan combinations:
  • Planning values: \( n_t = 20-60 \) (5 values), \( \text{Pr}\{\text{ZFP1}\} = 0.5\%-20\% \) (5), \( \sigma = 0.6-2.0 \) (6), \( \text{pf}_{U} = 0.0005-0.01 \) (3), \( \text{pf}_{H} = 0.45-0.95 \) (4), \( \eta_{U} = \eta_{c} \)
  • Life distribution: Weibull & Lognormal
  • Other input values, normalized test duration at each stress:
    \( \eta_{L}/\eta_{c} = 1, \eta_{M}/\eta_{c} = 1 \& 0.75, \eta_{H}/\eta_{c} = 1 \& 0.5 \)
  • Calculate minimum \( \xi_{L} \) and the corresponding fractional allocations \( \pi_{L\text{min}}, \pi_{M\text{min}} \)
  • Use MC simulation to find the (single) optimal allocation, corresponding to the minimum Var
  • Look for possible correlation between the optimal \( \pi_{L\text{opt}}, \pi_{M\text{opt}} \) and the input values. The optimal allocations are unknown, unless MC is done.
Useful Results: Parameter Space Reduction

• Selection of $n_t$, Pr{ZFP1} and planning values, to obtain $\xi_L$ in the range 0.3-0.8

• For equal plan ($\eta_U$ & $\eta_H$) and test ($\eta_L$, $\eta_M$, $\eta_M$) durations, compared to unequal shorter durations:
  • $\min(\xi_L)$ & min Var are smallest, and do not depend on $\sigma$
  • $\min(\xi_L)$ & min Var depend on the normalized plan & test durations, not the absolute values, e.g. 1:1 & 1:0.75:0.5

• The allocation ($n_L$, $n_M$, $n_H$) corresponding to the $\min(\xi_L)$ does not depend on the planning $pf_U$, only the $pf_H$
  • also appears to be true for the optimal allocation corresponding to min Var, within ±2 units
Correlation with Optimal $\pi_L$ & $\pi_M$

Note outliers
Outliers Caused by Multiple Minima

Var saddle response surfaces typically occur for $\Pr\{ZFP1\}>10\%$ & Weibull life

Planning values:
- $n=40$
- $p_{f_U}=0.001$, $p_{f_H}=0.9$
- $\sigma=0.6$
- $\eta_c = \eta_L = \eta_M = \eta_H$
- $\Pr\{ZFP1\}=20\%$

Var saddle response surfaces typically occur for $\Pr\{ZFP1\}>10\%$ & Weibull life
Model Prediction for Optimal $\pi_L$ & $\pi_M$

Model: $\pi_{L\text{opt}} \sim \xi_L + \pi_{L\text{min}}$  $R^2 = 0.916$

$\pi_{M\text{opt}} \sim n_t + \text{PrZFP1} + \pi_{L\text{min}} + pf_H$  $R^2 = 0.932$
Attributes of Proposed Method

• Automates Ma & Meeker method (steps 1 & 2) to identify candidate test plans
• Requires only the constraint of equally spaced stress levels. Not equal failures for levels L & M
• Does not use the large sample approximation, Avar
• Shows the dependence between Pr\{ZFP1}, ξ\_L and allocation
• Enables different censor times for each stress level
• Can be extended to 4 or more stress levels, 2 stress factors
• Monte Carlo simulation (step 3), is recommended to fine tune the allocation of candidate test plans

**Key question:** how good are the model plans?
Model Test Plan Comparisons

• Model allocations are generally within ±2 units of the optimum, corresponding to min Var

• In addition to the high Pr\{ZFP1\} case, large discrepancies were found when $\xi_L > 0.8$, with steep Var response surface. $\xi_L$ can easily be lowered.

Planning values:

$n_t = 30$
$p_f_U = 0.01$, $p_f_H = 0.75$
$\sigma = 0.75$

$\eta_U = \eta_H$;
$\eta_M : \eta_M : \eta_H = 1 : 0.75 : 0.5$

$Pr\{ZFP1\} = 1%$

<table>
<thead>
<tr>
<th>$\xi_L$</th>
<th>Var</th>
<th>$n_t/n_f$</th>
<th>$n_t/n_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8194</td>
<td>430.13</td>
<td>33.3%</td>
<td>40.0%</td>
</tr>
<tr>
<td>$&gt;1.0$</td>
<td>unknown</td>
<td>30.0%</td>
<td>33.3%</td>
</tr>
</tbody>
</table>

$\eta_U/\eta_L = 1 : 0.75 : 0.5$

Var not determined for $\xi_L > 0.95$
Conclusions

• The method simultaneously optimizes the stress levels and allocation, for input $n_t$, $Pr\{ZFP1\}$ and planning estimates, to minimize the variance of the extrapolated B10 life at Use
• The regression model estimate of the optimal allocations, based on $\min(\xi_L)$ and the corresponding fractional allocations, are generally within ±2 units of the Monte Carlo simulation
• Large discrepancies occur for relatively few cases; at high $Pr\{ZFP1\}>10\%$ and $\min(\xi_L)>0.8$ (latter easily mitigated)
• The method gives higher allocation to L & H, particularly for $n_t>30$, significantly different from traditional fixed 4:2:1 allocations
• The optimal allocation gives the best chance of minimizing the error of the estimated time to failure
Thank you for your interest

Questions?
Backup slides

• Other optimization methods
• Prediction model statistics & coefficients
Previous Test Plan Optimization

• Ma & Meeker recommend a 3-step method:
  1) Use formulae to determine the region of $\xi_L$, $\pi_L$ & $n_t$ for which $Pr\{ZFP1\}$ meets a target value, e.g. 1%
     For simplified calculation M&M imposed a 2\textsuperscript{nd} constraint of equal number of failures at levels L & M: $n_L pf_L = n_M pf_M$
  2) Find a tentative test plan with allocation that minimizes $(n_t / \sigma^2).Avar(y_p)$ and achieves target $Pr\{ZFP1\}$
     Asymptotic variance "Avar" is the large sample approximation
  3) Use Monte Carlo simulation to fine tune the tentative plan and determine the actual variance $(n_t / \sigma^2).Var(y_p)$

• Avar requires computation of the Fisher information matrix
• Avar may not be a good approximation of variance for n<50
SAS JMP: ALT Plan Optimization

**Design**

<table>
<thead>
<tr>
<th>X1</th>
<th>N Units</th>
<th>Expected Failures</th>
<th>All Censored</th>
<th>Failure Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>91.03</td>
<td>12</td>
<td>1.5</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>104.96</td>
<td>16</td>
<td>6.8</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>12</td>
<td>10.9</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>

**Parameter Variance for Optimal Design**

**Optimality Criteria**

- Quantile Estimate Optimal Design: -1.161
- Quantile Criterion: 0.4566
- Probability Criterion: 4.7662

**R Precision Factor (95% CI)**

- 9.319524
- 9.319524
- 9.319524

**Simulator**

- Number of Simulations: 10000
- Simulation Probability of Interest: 0.1
- Simulation Usage for X1: 50

**Run Simulations**

- Average: 17.05
- SD: 183.9
- 0.1 Log Quantile: 8.185
- R Precision: 1.25
- Intercept: 17.05
- X1: 6.862
- scale: 0.748
- 0.229
- 0.121
Using JMP for ALT Optimization

- Compare plans with consistent \( \text{Pr}\{ZFP1\} \)
- JMP optimizes the allocation for given stress conditions
- JMP gives the All Censored Probability, but not used for optimization
- Optimality Criteria: Quantile, Probability & R precision, correlate with Var
- Optimization method is iterative:
  - Change \( n_M \), find optimal allocation for given \( n_t \) by Monte Carlo simulation
  - Adjust stress levels to achieve target \( \text{Pr}\{ZFP1\} \)
  - Rerun simulation, allocation usually changes
  - Find the allocation corresponding to the overall best relative optimality criterion
Model for Optimal $\pi_L$

```r
> summary(mod2)  # Data points 5 & 9 excluded
Call:
lm(formula = AL1 ~ xIL2 + AL2)

Residuals:
   Min     1Q   Median     3Q    Max
-0.06021153 -0.01241463  0.00063236  0.01453606  0.05854014

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)  
(Intercept)        0.3361249  0.0432599  7.76990 1.2120e-11 ***
xIL2               -0.1491961  0.0307095 -4.84882 5.1661e-06 ***
AL2                0.5381618  0.0422528 12.73673 < 2.22e-16 ***
---
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.0234532 on 90 degrees of freedom
Multiple R-squared: 0.9239, Adjusted R-squared: 0.9239
F-statistic: 501.151 on 2 and 90 Df, p-value: < 2.22e-16

> AIC(mod2)
[1] -429.138108
```
Model for Optimal $\pi_M$

```r
> summary(mod3)
Data points 5 & 9 excluded
Call:
lm(formula = AM1 ~ nt + PrZF1 + AL2 + PrfH)

Residuals:
  Min 1Q Median 3Q    Max
-0.04667710 -0.00935125  0.00244473  0.01332476  0.04904097

Coefficients:
                        Estimate Std. Error  t value Pr(>|t|)  
(Intercept)            0.619655551 0.013825450 44.81992    <2.2e-16 ***
nt                      -0.001297559 0.000303558  -4.27482  4.335e-05  ***
PrZF1                   -0.394016459 0.043461422  -9.06589     3e-14  ***
AL2                     -0.431579118 0.036000830 -11.98803     <2.2e-16  ***
PrfH                    -0.125713202 0.019368427  -6.42560  1.066e-08     ***

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.0191517 on 88 degrees of freedom
Multiple R-squared: 0.935129, Adjusted R-squared: 0.932181
F-statistic: 317.137 on 4 and 88 DF, p-value: < 2.2e-16

> AIC(mod3)
[1] 464.914962
```
Correlation with Optimal $\xi_L$
Model Prediction for Optimal $\xi_L$

Model: $\xi_L^{\text{opt}} \sim n_t + \text{PrZFP1} + \xi_L^{\text{min}} + \text{pf}_H + \left(\frac{\eta_H}{\eta_U}\right)$ \quad R^2 = 0.979

Note: optimal $\xi_L$ shown for reference, in practice this is determined from the model optimal allocations.
Model for Optimal $\xi_L$

```
> summary(mod1)  # Data points 5 & 9 excluded
Call:
lm(formula = x1L1 ~ nt + PrZFp1 + x1L2 + PrfH + RetaH)
Residuals:
  Min     1Q    Median     3Q    Max
-0.03526213 -0.01125008   0.00286164   0.00794551   0.07510139
Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)  -0.283165232   0.040244846   -7.03609  4.2943e-10    **
              -0.283165232
nt          0.002164329   0.000267843    8.08058   3.3940e-12    ***
              0.002164329
PrZFp1       0.272316789   0.045349812    6.00481   4.3228e-08    ***
              0.272316789
x1L2        1.065980600   0.023961615   44.48724  < 2.22e-16    ***
              1.065980600
PrfH        0.173423432   0.019122407    9.06912   3.2420e-14    ***
              0.173423432
RetaH       0.052399597   0.010587466    4.94921   3.6112e-06    ***
              0.052399597

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.0193123 on 87 degrees of freedom
Multiple R-squared: 0.579822, Adjusted R-squared: 0.578662
F-statistic: 844.919 on 5 and 87 DF,  p-value: < 2.22e-16

> AIC(mod1)
[1] 462.423912
```